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A mixing criterion for turbid rivers

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Abstract

A criterion for the formation/destruction of thermal stratification in turbid rivers is developed and tested against an extensive data set in a weir pool on the Murrumbidgee River, Australia. The criterion estimates whether a river section would stratify or not, subject to a prescribed heat flux through the water surface and a prescribed river flow. It can be determined from readily accessible parameters, including solar radiation, turbidity, wind speed and river discharge.

It is suggested that this simple mixing criterion be used as a flow management tool in a variety of water quality applications in rivers. © 1998 Elsevier Science Ltd. All rights reserved

Keywords: Stratification; How management; Solar radiation; Wind speed

1. Introduction

Low flow conditions combined with high solar radiation often lead to persistent thermal stratification in lowland Australian rivers. Low flow is often put forward as a stimulatory factor for the growth of cyanobacteria in turbid rivers. Flow was found to be a key factor in the regulation of phytoplankton abundance and species composition in the Murray River (Sullivan *et al.*, 1988), the Darling River (Hötzel and Croome, 1994) and the Murrumbidgee River (Jones, 1993).

Data collected over a three year period in Maude Weir pool on the Murrumbidgee River, 50 km west of Hay, NSW, Australia (Fig. 1), demonstrate that persistent stratification favours the occurrence of blooms of the toxic cyanobacterium *Anabaena circinalis*. Blooms did not occur when river discharges were high enough to cause complete mixing of the water column on a diurnal basis (Sherman *et al.*, 1994). Management of cyanobacterial blooms could be achieved by manipulating the size and timing of the discharge in such a way that thermal stratification is destroyed every few days (Webster *et al.*, 1996). Based purely on data collected over two years in the Murrumbidgee River, Jones (1993) determined empirically a relationship between the observed temperature stratification, the river discharge versus depth, and the solar radiation.

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In this paper, a non-dimensional parameter for predicting the transition between mixed and stratified conditions in turbid rivers is presented and tested in Maude Weir pool. The criterion is based on that first developed by Simpson and Hunter (1974) for estimating whether a coastal sea would stratify or not subject to a prescribed magnitude of tidal current and a prescribed heat flux through the water surface. This was later extended by Holloway (1980) to include penetration of solar radiation into the water column.

2. Mixing criterion

The degree of stratification in a water column is determined by the relative supply rates of stratifying thermal energy and destratifying turbulent kinetic energy. A negative buoyancy flux (surface heating) decreases the potential energy (PE), while mixing, resulting from turbulent kinetic energy, increases it. In a river the thermal energy is supplied mainly by the sun, but this can be offset by night-time cooling, evaporative fluxes and long wave radiation, whereas the turbulent kinetic energy is derived mainly from the wind blowing on the water surface and from the flow over the bottom.

Following the approach of Simpson and Hunter (1974) and Holloway (1980), the rate of change of the potential energy (PE) required to mix the heat input and maintain a well-mixed water column of depth H, heated by a net surface heat flux Q_{net} is given by

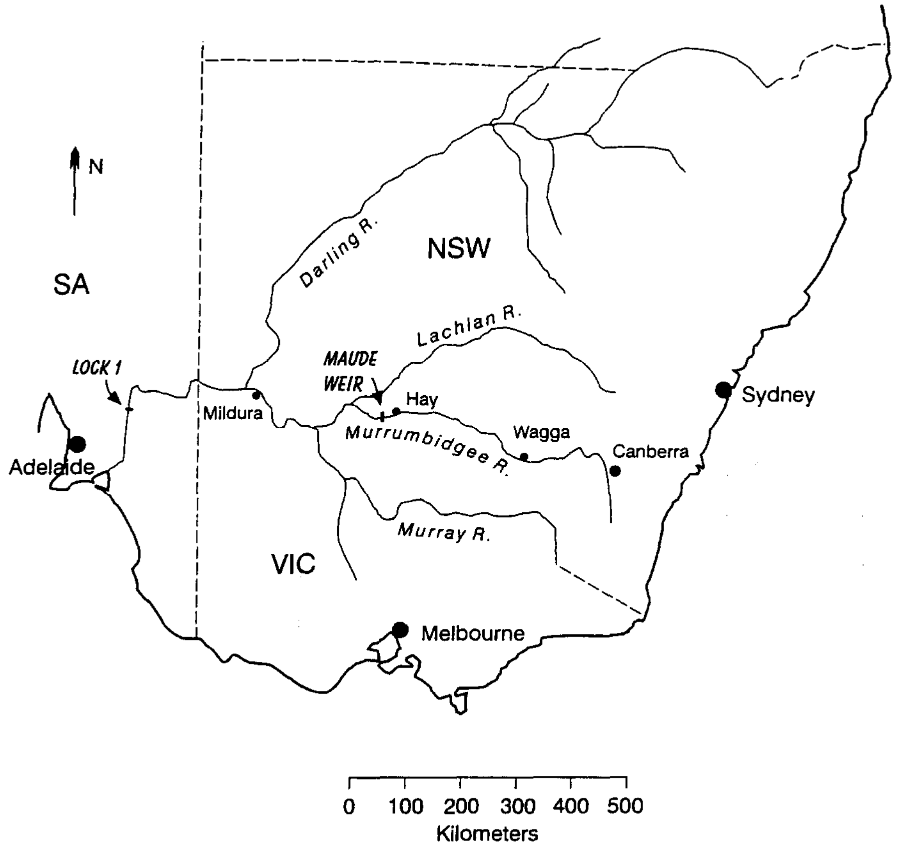

Fig. 1. Map of south-eastern Australia showing the major rivers of the Murray-Darling Basin and the location of the field experiments in Maude Weir pool and below Lock 1.

$$\frac{d(PE)}{dt} = \frac{\alpha g H}{2C_p}\left(Q_{net} - \frac{2Q_l}{K_d H}\right) \tag{1}$$

where α is the thermal expansion coefficient, g is the gravity acceleration, C_p is the specific heat capacity of water, Q_l is the net short wave radiation and K_d is the attenuation coefficient. In deriving this formula, it is assumed that the solar radiation penetrating the water column is attenuated exponentially with depth by turbidity and water colour and that there is no heat loss through the bottom of the water column. The net surface heat flux Q_{net} can be expressed as the sum of the radiative (short and long wave radiation), evaporative and sensible heat fluxes where the individual fluxes can be estimated from bulk formulations (see Appendix) using wind speed, air and surface water temperature, and relative humidity.

The input heat flux resulting from a horizontal temperature gradient will be neglected on the basis that in a river the water will quickly adjust to its equilibrium temperature with the atmosphere. Indeed, the surface water temperature is limited by a feedback mechanism in which the warmer the surface layer gets, the faster it looses heat through evaporation, conduction to the atmosphere, and emission of long wave radiation. This is in contrast to estuarine stratification where horizontal density gradients are considerable (Nunes and Lennon, 1987; Simpson *et al.*, 1990).

In developing this mixing criterion for turbid rivers, the flow over the bottom will be assumed to be the only source of turbulent kinetic energy. Wind mixing will be neglected on the basis that the measured temperature profiles in two different rivers of various width and exposition showed a consistent very shallow surface mixed layer ($\leq$ 20 cm) during the day when winds were strongest. However, wind will be important in determining the evaporative and sensible heat fluxes which contribute significantly to the net surface heat flux. The rate of working against bottom friction will be given by

$$\frac{d(TKE)}{dt} = c_D \rho_w U^3 \tag{2}$$

where c_D is the bottom friction coefficient which applies to the depth-averaged velocity U, and ρ_w is the density of water. A fraction ϵ of this *TKE* is available for increasing the *PE* of the water column. The water column will destratify if the rate of increase of *PE* due to turbulent mixing exceeds the rate of decrease of *PE* due to surface heating. When the two competing processes are equal the transition between mixed and stratified conditions can be expressed as

$$R = \frac{U^3}{H\left(Q_{net} - \dfrac{2Q_I}{K_d H}\right)\dfrac{\alpha g}{\rho C_p}} = \frac{1}{2\epsilon c_D} \tag{3}$$

It is important to note that this criterion is not valid when the expression in parentheses is negative, which is when the water column is cooling. In that case, the water column will stay well mixed regardless of the discharge.

3. Application of the mixing criterion

Data from the Maude Weir pool were used to test the mixing criterion. One of the major inland rivers of Australia, the Murrumbidgee is characterised by relatively high turbidity (10–20 NTU) and low bed slope (1:11,000) as it flows across the Hay Plain. The average river width is 40 m and the average depth is about 5 m. An extensive collection of physical, chemical and biological data was undertaken in the weir pool to determine the environment conducive to cyanobacterial blooms (Webster *et al.*, 1996). During the summers of 1993–1994 and 1994–1995 a meteorological station was installed on a raft to measure a number of parameters liable to affect the stratification behaviour of the weir pool. These included wind speed and direction, solar radiation, air temperature, and humidity. Three thermistor chains, each consisting of 19 thermistors placed in the vertical, were deployed at the same time. All data was recorded at 10 min intervals except the discharge which was recorded daily at the weir. Large variations in discharge between 150 ML d^{-1} and 5200 ML d^{-1} were experienced during the summer periods.

The parameter R was calculated at hourly intervals from measured values of Q_I and river discharge and values of Q_{net} inferred from measured wind speed, air temperature and humidity (see Appendix). Although these values are based on instantaneous measurements, they are slowly varying quantities with a similar time response to changes. During the night when the water column was cooling, the data were not used as R was negative. R is plotted as a function of discharge in Fig. 2. Stratified and well mixed conditions have been distinguished on the basis of the thermistor measurements. The water column was taken to be stratified if the temperature difference between the top and bottom thermistors was equal or greater than 0.05°C. Well mixed conditions in the weir pool generally corresponded to $R > 55{,}000$, while stratified conditions corresponded to $R < 35{,}000$. A few exceptions can be identified in Fig. 2. However, these were characterised by temperature differences of around 0.05°C between the top and bottom thermistors, which is at the limit of distinguishing between stratified and mixed conditions. Data corresponding to the stratification behaviour of a section of the lower River Murray just downstream of Lock 1 (Fig. 1), investigated by Bormans *et al.* (1997) have also been plotted in Fig. 2. Although these data do not cover a significant range of parameter values, they provide other evidence to support the applicability of the mixing criterion.

In order to narrow down the value of R corresponding to the transition between well mixed and stratified conditions, a numerical model was used to test Eq. (3) and to determine the value of ϵ. The model is a two-dimensional version of the Princeton Ocean Model used by Bormans and Webster (1998) to reproduce the mixing regimes of a riverine channel in response to river discharge and meteorological forcing. The model simulates the turbulent mixing and advection of momentum and heat through a river section of constant depth and assumes no gradient across the width of the river. The surface heat flux Q_{net} is prescribed together with the solar radiation Q_I which is attenuated exponentially with depth. Uniform flow was assumed at the upstream and downstream end of the river section considered. For a given surface heat flux, the model was run at increasing discharges until the stratification was totally destroyed. To predict the transition under a wide range of conditions, the depth of the water column was varied from 3 to 10 m in the model runs.

Fig. 2 shows that the non-dimensional number R predicted by the model for the transition between stratified and mixed conditions is approximately a constant regardless of the discharge as Eq. (3) suggests. The model predictions show that the transition between well mixed and stratified conditions in rivers takes place around $R = 45{,}000$. This value is consistent with the measurements in Maude Weir pool. Using a typical value for the bottom friction coefficient of $c_D = 2.5 \times 10^{-3}$ (Csanady, 1982), the predicted value of R for the transition gives a mixing efficiency $\epsilon = 0.0044$, which is close to the value of 0.0037 estimated by Simpson and Hunter (1974) for coastal seas using the same bottom friction coefficient. This is encouraging but not surprising as both criteria are based on mixing generated by bottom friction.

4. Discussion

A criterion to assess mixing conditions in turbid rivers has been determined from readily available parameters such as river discharge, solar radiation, depth of light penetration, and wind speed. The criterion was tested in a weir pool of a turbid river in Australia from detailed measurements of the thermal structure under a wide range of river discharges and meteorological forcing. A numerical model was also used to define more precisely the transition estimated from the data and thereby determine the mixing efficiency. The transition predicted by the model suggested that the fraction ϵ of the kinetic energy available for mixing

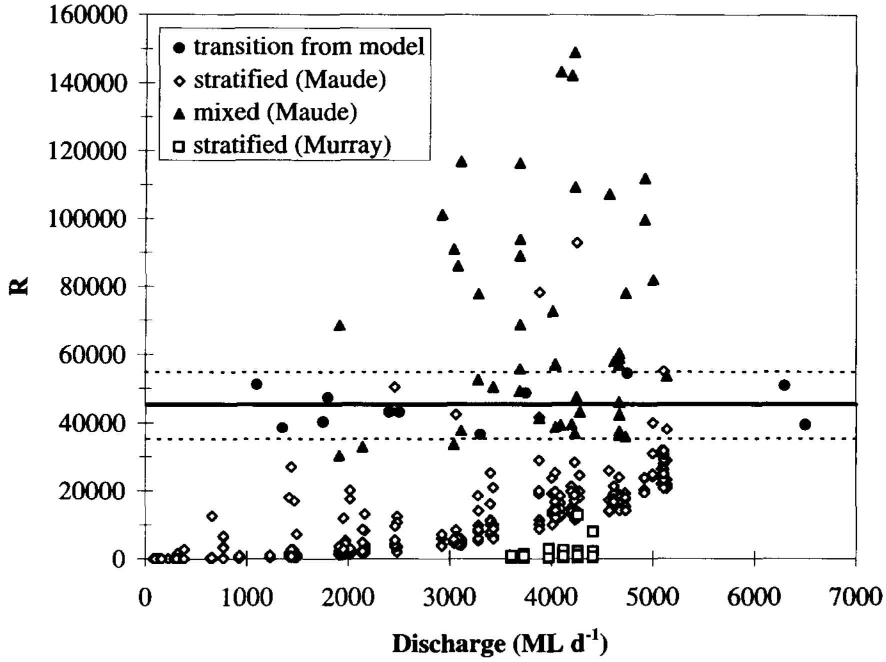

Fig. 2. Non-dimensional parameter R versus river discharge. A line is drawn at R = 45,000 to represent the averaged location of the transition based on both the observations from Maude Weir pool and the model predictions. Field data from the lower River Murray representing stratified conditions have also been included.

in rivers was similar to that predicted by Simpson and Hunter (1974) for mixing in coastal regions due to tidal forcing.

This simple classification scheme can be used in developing management strategies to address water quality problems in river systems. If applied to daily average quantities, the criterion predicts whether a water column is likely to be persistently stratified (some stratification remaining throughout both the day and night) or whether it will mixed diurnally (stratification disappears some time during the 24 h period). The implications of whether or not a river section remains stratified or mixes diurnally are important in a wide range of applications. We have already shown how persistent stratification can significantly enhance the growth of toxic cyanobacteria, resulting in blooms, as the floating algae are kept in a light environment favourable for growth (Sherman *et al.*, 1994). Persistent stratification by significantly decreasing vertical mixing will also restrict the vertical redistribution of nutrients and oxygen through the water column, leading to bottom anoxic waters which potentially enhance the release of nutrients or other chemicals from sediments.

5. Appendix

The net surface heat flux Q_{net} into the water column is given by

$$Q_{net} = Q_I - Q_B - Q_s - Q_E$$

where Q_I is the net downward short wave radiation, Q_B is the net upward long wave radiation, Q_s is the upward sensible heat flux and Q_E is the heat flux of evaporation.

• The net downward short wave radiation under clear skies can be calculated as a function of latitude and time of day as

$$Q_I = Q_{Io}(1 - \alpha_S)\sin\beta$$

where the maximum radiation Q_{Io} = 1000 Wm^{-2}, the surface albedo α_s = 0.1, and β the solar altitude or elevation is given by

$$\sin\beta = \sin\phi\,\sin\delta + \cos\phi\,\cos\delta\,\cos h.$$

In the previous expression, ϕ is the latitude of the location (positive in the Northern Hemisphere and negative in the Southern Hemisphere), δ is the solar declination and h is the angle hour. The solar declination depends on the day of the year and is expressed as

$$\delta = -\,23.4\cos[360(t_j + 10)/365]$$

where t_j is the Julian date in the year and h is related to the local apparent solar time t by

$$h = 15(12 - t).$$

The net short wave radiation crossing the water surface is attenuated exponentially with depth by turbidity and water colour following Beer's law

$$I(z) = Q_I e^{K_d z}$$

where K_d is the attenuation coefficient in m^{-1}, and z is zero at the water surface (positive upwards). For Maude Weir pool, Q_I was measured and K_d was inferred from measured light profiles.

• The net upward long wave (thermal) radiation Q_B is absorbed and emitted from within the top millimetre of the water surface. It can be estimated in the absence of clouds by

$$Q_B = e\sigma(T_s + 273.16)^4(0.39 - 0.05e_a^{1/2})$$

where T_s is the water surface temperature (°C), $e = 0.972$ is the emissivity of the water (Henderson-Sellers, 1986) and $\sigma = 5.7 \times 10^{-8}$ $Wm^{-2}\,°K^{-4}$ is the Stefan–Boltzmann constant. The last factor accounts for back radiation and depends on the vapour pressure of water e_a. For Maude Weir pool, the net upward long wave radiation was measured.

• The upward sensible heat flux is obtained from

$$Q_s = C_p \rho_a C_H W(T_s - T_a)$$

where C_p is the specific heat capacity of air, C_H is a dimensionless exchange coefficient for sensible heat exchange, W is the measured wind speed (m s^{-1}) and T_a is the air temperature (°C).

• Evaporation is the most important mechanism causing heat loss from the water column. The heat flux due to evaporation Q_E, or latent heat flux, causing heat loss from the water column is given by

$$Q_E = L_v \rho_a C_E W(q_s - q_a)$$

where L_v is the latent heat of evaporation, ρ_a is the density of air, C_E is a dimensionless exchange coefficient for evaporative heat exchange, q_s and q_a are the specific humidities estimated from surface water temperatures, air temperature and humidity using the functional form of Kimball *et al.* (1982).

The bulk formulae used for the evaporative and sensible heat fluxes are typical of those used in oceanic or open waters applications (Kraus, 1972; Henderson-Sellers, 1986). However, the values of the coefficients C_E and C_H were adjusted so that the sum of the radiative, evaporative and sensible heat fluxes equalled the average Q_{net} estimated from closing the heat budget for Maude Weir pool. This gave $C_E = 3 \times 10^{-3}$ and $C_H = 2 \times 10^{-3}$, which were about twice the conventional values for open waters (Smith, 1980). While these values were determined exclusively from Maude Weir pool data, they also gave the best predictions for the stratification behaviour in the lower River Murray under a wide range of discharge and meteorological forcing.

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