Postoptimality Analysis of an Ecosystem Management Simulator

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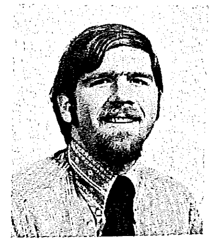
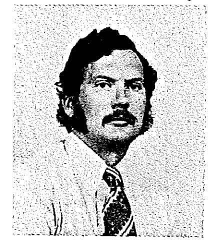
CHAPTER 11

Postoptimality analysis of an ecosystem management simulator





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ABSTRACT

This paper describes a computer-implemented simulation model of the Rich Creek stream fishery in West Virginia; the model, however, is general enough to be adaptable to the management of any aquatic ecosystem. Major features of the model are (1) a stochastic process to quantify interactions between and among fish species, (2) a dynamic equation of angler density as a function of time of year, weather, previous success, and anticipation, and (3) a measure of satisfaction ("utility") for recreational anglers. The measure of utility is an objective function which is maximized by an optimization algorithm subject to constraints on maximum budget, minimum commercial catch, and minimum diversity of fish species. When an optimal solution has been found, it is subjected to a postoptimality analysis which shows the effects of small perturbations of variables on the three constraints and other parameters. Results show that high diversities of fish species and high commercial catches cannot occur simultaneously. At low levels of budget for managing the stream fishery the diversity constraint does not limit utility; diversity is already at a fairly high level. Temperature and metabolic rate are important variables in the model; that is, perturbations in these variables produce relatively large changes in utility. The size of the human population with access to the stream and the size of the management budget also significantly affect utility. Models of ecosystems cannot be completely general, and must be designed to "answer" specific questions. Optimization of course requires an explicit objective function, which is also needed to judge the sensitivity data obtained from the postoptimality analysis.

Keywords: angler satisfaction, optimal control theory, search by simulation, stochastic interaction, utility function

INTRODUCTION

Computer simulation has become a popular tool for ecological study, and its potential for scientific and management applications to ecology is considered to be great.[1,2] But, complex interactive computer-implemented models have been criticized because the hypotheses about parameter values and logical patterns used in the models are not often tested.[3] One method available to test these hypotheses is sensitivity analysis. The sensitivity of simulation solutions to small changes in parameters of the model is usually expressed as partial derivatives of state variables with respect to system components.[4] One type of sensitivity analysis, often used in operations research, is the study of the behavior of the optimal solution when small changes are made in model variables.[5] Such studies are termed *postoptimality analysis*; they require use of an explicit measure of system performance (an objective function) which is maximized or minimized subject to constraints.

It is unlikely that a scientist is going to accept a theory or an ecosystem manager is going to accept a decision policy on the basis of a single simulation experiment. Fortunately analysis of simulation results provides insight into the likely dynamic structure of the ecosystem and into the tradeoff effects of multiple factors, such as exploitation of the resource by competing users versus the limited ability of the ecosystem to maintain itself under differing conservation strategies. This paper reports a post-optimality analysis of a model of a stream ecosystem. The model was designed to investigate management policies for fisheries resources.

Table 1

Common names, scientific names, and simulation category names for species in Rich Creek

Common name	Scientific name	Category name
Particulate organic matter		POM
Macroinvertebrates		MACROINV
Rainbow trout	*Salmo gairdneri*	TROUT
Brown trout	*Salmo trutta*	
Brook trout	*Salvelinus fontinalis*	
Bluegill sunfish	*Lepomis spp.*	BLUEGILL
Rock bass	*Ambloplites rupestris*	
Smallmouth bass	*Micropterus dolomieui*	SMOUTHBS
Spotted bass	*micropterus punctulatus*	
Crayfish	*Cambarus spp.*	CRAYFISH
Crayfish	*Orconectes spinosus*	
White shiner	*Notropus albeolus*	SHINERS
Telescope shiner	*Notropus telescopus*	
Greenside darter	*Etheostoma blennioides*	
Fan-tail darter	*Etheostoma flabellare*	
Bluehead chub	*Hybopsis leptocephala*	CHUBS
Creek chub	*Semotilus atromaculatus*	
Mottled sculpin	*Cottus bairdi*	
Stone roller	*Campostoma anomalum*	STONERLS
Bluntnose minnow	*Pimephales notatus*	
Fat head minnow	*Pimephales promelas*	
White sucker	*Catostomus commersoni*	SUCKERS
Northern hogsucker	*Hypentelium nigricans*	

SIMULATION MODEL

The stream fishery modeled is patterned after a 10-mile section of Rich Creek, Monroe County, West Virginia. Components of the ecosystem which are exploited include (1) a heavily fished rainbow-trout population supported entirely by stocking with catchable-size trout, (2) bluegill and smallmouth bass populations for which there is a relatively small amount of fishing, and (3) baitfish species which are exploited by commercial fishermen using seines and traps. Table 1 summarizes common, scientific, and category names (into which each species was placed for this simulation) for the Rich Creek ecosystem. All fish but trout, bluegill, and smallmouth bass are treated as bait species.

The details about the development and structure of the model are given by Powers[6] and will not be discussed here. However, we present the following description to show the general form of the equations, the components (categorized fish species) of the equations, and how the variables interact.

The simulation model is composed of difference equations from which numerical results are calculated at specified points in simulated time. The simulation results reported here are from runs of 24 time periods of 15 days each (approximately 1 year). We estimated the parameters and constants for the functions subjectively or, if data were available, by iterative least-squares regression.

Interaction between and among the fish species is modeled using a stochastic process in which an animal's actions are governed by a preemptive priority sequence: detection of a high-priority stimulus (arrival) will force the animal to stop behavior toward (service of) a stimulus of a lower priority; that is, high-priority stimuli *preempt* service of lower priority stimuli.

Given m priorities $(1,2,\ldots,m$, with high priorities first) then P_{oi}, the steady-state probability of the animal's being idle (not acting on a stimulus) as a result of not having been interrupted during a service of the ith priority, is:

$$P_{oi} = \frac{\mu_i \gamma_i}{\left(\sum_{j=1}^{m} \lambda_j\right)\left(1 + \sum_{j=1}^{m} \gamma_j\right)} \quad (1)$$

where

μ_i = the service rate, i.e., the number of services of type i that the animal is physically capable of accomplishing per unit time

λ_i = the arrival rate, i.e., the number of stimuli of type i that the animal detects per unit time

and where γ_i are found recursively from m to 1 by

$$\gamma_i = \frac{\gamma_i\left(1 + \sum_{j=i+1}^{m} \gamma_j\right)}{\mu_i + \sum_{j=1}^{i-1} \lambda_j} \quad (2)$$

P_{oi} is the proportion of time that the animal is idle, i.e., that it has completed an activity (service) of priority i without interruption by a service or higher priority. Of these noninterrupted acts, a certain proportion β is completed successfully. $\beta = \mu_i/(\mu_i + \mu_i^{\sim})$, where $\mu_i^{\sim}$ is the complementary service of μ_i. For example, if μ_i is the rate of capturing prey z by predator y, then $\mu_i^{\sim}$ is the rate at which prey z escapes from predator y.

The reciprocal of $P_{oi}\beta$ is the expected amount of time between successful completions of a service of priority i. This reciprocal divided into the total time of interaction is the number of successful completions (X_i) during that time of interaction.

Similarly, the reciprocal of $P_{oi}(1-\beta)$ divided into total time is the number of *unsuccessful* completions (Y_i) during the same time of interaction.

Arrival rates λ_i are calculated by

$$\lambda_i = 2DR_iN_r\left(S_v^2 + S_r^2\right)^{\frac{1}{2}} \quad (3)$$

where

D = surrounding area in which animal v can detect stimuli

N_r = densities of species r

S = speeds with which animals v and r move in the ecosystem

R_i = proportion of species r that are in the ith priority.

R_i is time-, species-, and size-dependent in the simulation model, and the speed S is a nonlinear function of temperature, fish size, and fish species.

The priority sequence (i = 1,2,...,6) employed in the model is (with high priorities first):

(a) Escape from a predator (i=1)

(b) Escape from a dominant aggressor (i=2)

(c) Reproduce with a receptive mate (i=3)

(d) If in reproductive condition, attack a subdominant fish of the same or different species (not for the purpose of eating it) (i=4)

(e) Chase a prey (i=5)

(f) If not in reproductive condition, attack a subdominant (i=6).

Table 2

Decision activities expressed explicitly in the simulation model

Decision number	Definition
1	Fraction of total area that is open for fishing
2	Biomass TROUT (grams/m^2) to be planted
3	Mesh size of seine or trap (cm)
4	Maximum length of seine (m)
5	Maximum number of baitfish in possession
6	Density of commercial fishermen (number/m^2)
7	Maximum number of TROUT in possession
8	Minimum size of BLUEGILL (grams) in possession
9	Maximum size of BLUEGILL (grams) in possession
10	Maximum number of BLUEGILL in possession
11	Minimum size of SMOUTHBS (grams) in possession
12	Maximum size of SMOUTHBS (grams) in possession
13	Maximum number of SMOUTHBS in possession

Priorities i = 4, 5, and 6 imply that reproduction and associated social behavior generally take precedence over feeding behavior, which is priority (e). This appears to be true for many fishes.[7]

The numbers of successful and unsuccessful completions of activities per time period, Y_1, X_3, and X_5 (where the subscripts refer to priority i = 1, 3, and 5, respectively), are especially important, and they are calculated using (1), (2), and (3). The mortality rate (Y_1), natality rate (X_3 times fecundity), and predation rate (X_5) are determined for each age-class in each species. Fecundity is a nonlinear function of fish species, fish size, time of year, and temperature. Ration size (weight of prey consumed per animal) is found by multiplying X_5 by the weight of the prey. Growth per individual is then calculated as a nonlinear function of ration size, temperature, and fish size. Particulate organic matter density, macroinvertebrate density, and temperature are functions of time only and are, therefore, forcing functions for the stochastic interaction model and its associated submodels. The stochastic model is iterated over each age-class in each species.

The decision variables (those under management's control) govern the rate and kind of exploitation by commercial and recreational fishermen (Table 2). These decisions are time-dependent inputs in the model; there are 13 decision activities in each of 24 time periods or 312 decision-variable inputs.

The density of commercial fishermen may be controlled by the state agency and is an input decision variable (Decision 6 in Table 2). However, recreational fishing pressure (density of recreational fishermen) cannot be controlled directly by the agency. The number of people who fish during a time period depends on factors such as weather conditions, time of year, the success rate (catch rate) of fishermen during previous time periods, and anticipation due to a long time in which no fishing is allowed. This last factor is assumed to induce opening day crowds. Density of recreational fishermen (angler density) is predicted in the simulation model by the product

$$ANG = (POP)(PLIC(TC)(TY)(AN)(CA)/AREA \quad (4)$$

where

ANG = angler density (number/m^2)

POP = number of people with access to the stream

PLIC = proportion of POP with fishing licenses

AREA = surface area of the stream (m^2)

The other factors of (4) are

$$TY = EXP\ [-AP_1\ (\frac{TIME}{NSTAGE} - AP_2)^2]$$

$$TC = EXP\ [-AP_3\ (T - AP_4)^2]$$

$$AN = 1 - AP_5 EXP\ [-AP_6(OFF + 1)]$$

$$CA = 1 - AP_7 EXP\ [-AP_8(CPUE)]$$

where

TIME = time period

NSTAGE = number time periods in a year (=24)

T = temperature (°C)

OFF = number of consecutive previous days in which no fishing occurred

CPUE = catch-per-angler-day during the previous time period, and

AP_i = constants ($i = 1,2,\ldots,8$).

The density of anglers and commercial fishermen then become part of the stochastic interaction model; thus, catch rates by the fishermen depend on arrival rates, service rates, and the priority level given to the fishermen by the fish, i.e., escaping a seine would be priority one, while fish caught by traps or by hook are caught while in the act of feeding (priority 5). The catch of each age-class by recreational and commercial fishermen is then computed.

Management of an ecosystem implies that there is some specific objective or end sought by management. The management objective in this study is to maximize the benefits derived by recreational fishermen (anglers), subject to constraints on budget, on benefits derived by commercial fishermen, and on overexploitation.

Angler-derived benefits are estimated by a "utility" (objective) function. In this study the objective function is an index that includes the satisfaction that a consumer (angler) receives from alternative quantities of "commodities." Angler utility (satisfaction) in the simulation is a function of four independent variables or attributes:

(1) Species of fish caught — trout, bluegill, or smallmouth bass, or any combination of the three

(2) Average size of fish caught per angler — large fish preferred to small fish

(3) Average number of fish caught per angler — more fish preferred to fewer fish

(4) Crowding by other anglers (anglers/m^2/day) — less crowding preferred to more crowding.

Attribute 4 indicates that a party of fishermen fishing together will reduce the satisfaction experienced by each. This unrealistic feature is kept in the objective function because very little fishing in Rich Creek is done by parties of fishermen.

The attributes are found to be utility and preferentially independent, which allows the objective function to be developed using Keeney's model.[8,9] The contribution of each attribute to the overall utility is U_1, U_2, U_3, and U_4, where U is a nonlinear function of the four contributions. Questionnaires to obtain data for this study were sent to licensed fishermen living near Rich Creek. The responses to the questionnaires determined the shape and structure of the objective function. The range of this function is from zero to one. The objective function U for the simulation model is angler utility summed over all 24 time periods; i.e., the objective is to maximize U.

Three terminal constraints are included in the optimization problem. A terminal constraint is a constraint placed on the value of a state variable at the end of the last time period, i.e., at the end of the year. Budget B is terminally constrained so as not to exceed BDG dollars. The only variable costs in the model are those resulting from planting trout, and this cost is assumed to be \$0.0022 per gram of trout delivered to the stream (\$0.40 per average fish). Therefore budget B only reflects trout costs.

The second terminal constraint maintains a predetermined level of benefits derived by the commercial fishermen. Average commercial catch of baitfish per seining operation per day in each time period summed over all time periods is the measure of commercial benefits C. This measure is terminally constrained such that C is not less than COM fish.

The last terminal constraint is included so that not all fish will have been harvested at the end of the year. If a certain density of *each* of the fish species is to be maintained at the end of the year, the model would include eight more terminal constraints (one for each species), making computation difficult. However, some species have more "utility" to anglers and some have more important functions in the ecosystem than others do. Also, migration as a means of reintroduction of fish is quite likely in the stream. Since we desired to have a single constraint as an indicator of overharvest, we chose the diversity index:

$$\text{Diversity (in bits)} = 1.433\, \ell n \frac{N!}{N_1!N_2!\ldots N_n!} \qquad (5)$$

where

N = total number of individuals in the ecosystem

N_i = number of individuals in species i ($i = 1,2,\ldots,n$).

Trout are not included in diversity calculations because essentially all of the trout present are planted by the State of West Virginia. This index is a measure of "entropy,"[10] and the relationship between entropy and diversity may be rather tenuous.[11] However, (5) was chosen because it *does* depend upon the number of species n, the distribution N_i, and the total number N, each of which are important as indicators of overharvest. Therefore, diversity is terminally constrained so that at the end of the last time period, diversity D is not less than DV bits. The optimization problem may now be stated: Find the decision variables which maximize U subject to

$$B \leqslant \text{BDG}$$

$$C \geqslant \text{COM}$$

$$D \geqslant \text{DV}$$

Diversity in (5) is an abstract concept, and we had no *a priori* judgments about what DV should be. Also, we thought it possible that small perturbations of the constraints DV, COM, and BDG would alter the optimal value of U considerably. Therefore, a post-optimality analysis was conducted to determine the effect of uncertainties about the constraints and parameter values upon the optimal solution.

METHOD OF ANALYSIS

An approximate solution to the optimal-policy problem is found by use of a heuristic algorithm we designed specifically for simulation problems. The algorithm combines aspects of search by regression[12] and a policy-improvement algorithm utilizing the discrete maximum principle.[13]

Eleven indices (state variables) of system dynamics are used:

$$s_{tp} (t = 0,1,2,\ldots,25 \text{ and } p = 1,2,\ldots,11.)$$

The values of t are the 24 time periods that make up the year. The values of p have the following meanings:

1. Cumulative commercial catch per seine operation for each day (for 15 days per period)
2. Budget expenditures
3. Diversity index
4. Time of year (time-stage number t)
5. Biomass of POM and MACROINV
6. Biomass of TROUT
7. Biomass of BLUEGILL
8. Biomass of SMOUTHBS
9. Biomass of baitfish
10. Angler density
11. Cumulative sum of angler utility (satisfaction)

Next, k decision policies (d_{tq}) are randomly generated ($t = 1,2,\ldots,24$ time periods, and $q = 1,2,\ldots,13$ decision activities listed in Table 2). Using the k sets of d_{tq}, k simulation experiments are run and the s_{tq} and d_{tq} are recorded for each of the k experiments.

By linear regression analysis, transition functions (T_{tp}) are fitted to the k data sets

$$s_{t+1,p} = T_{tp}(\underline{s}_t, \underline{d}_t)$$

where $\underline{s}_t$ and $\underline{d}_t$ are the state and decision vectors, respectively. The models used for each T_{tp} are

$$s_{t+1,p} = T_{tp}(\underline{s}_t, \underline{d}_t) = \sum_{i=1}^{11} (a_i s_{ti} + b_i s_{ti}^2) + \sum_{j=1}^{13} (c_j d_{tj} + r_j d_{tj}^2) \qquad (6)$$

where $\underline{a}$, $\underline{b}$, $\underline{c}$, and $\underline{r}$ are state-dependent constants estimated by regression. A policy d_{tq} is optimal[6,13] if the stage-t Hamiltonian H_t

$$H_t = \sum_{p=1}^{11} \left[\theta_{t+1,p} \cdot T_{tp}(\underline{s}_t, \underline{d}_t)\right] \qquad (7)$$

is stationary with respect to $\underline{d}_t$. The state derivatives (θ_{tp}) are calculated recursively by

$$\theta_{tp} = \sum_{i=1}^{11} \left(\theta_{t+1,i} \frac{\partial T_{ti}}{\partial s_{tp}}\right) \qquad (8)$$

where the terminal state derivatives (i=25) are defined as

$\theta_{25,11} = 1$

$\theta_{25,i} = 0$; $i = 4,5,\ldots,10$ for the unconstrained terminal states

and where $\theta_{25,j}$ for the terminally constrained states ($j = 1,2,3$) are found using a decision inversion method.[13]

The H_t in (7) may also be expressed as

$$H_t = v_0 + \sum_{q=1}^{13} \left(v_q d_{tq} + w_q d_{tq}^2\right) \qquad (9)$$

where v and w are constants (constant functions of the stage-t state variables and state derivatives) because (6) is a quadratic function. Therefore, the stationary point at $(\partial H_t/\partial d_{tq}) = 0$ is

$$d_{tq} = -v_q/2w_q \qquad (10)$$

If d_{tq} from (10) exceeded bounds placed upon it ($0 \leqslant d_{tq} \leqslant d_{max}$), then d_{tq} is set equal to the bound exceeded (0 or d_{max}). Using (10), a new decision policy is derived for the fitted equations (6). A new simulation experiment is performed using the new decision policy and the search proceeds as follows:

1. If the value of the objective ("utility") function calculated from the new simulation experiment is greater than the lowest value of the objective function of the k previous simulation experiments, then replace the state and decision variables of the old experiment with the values obtained from the new experiment and go to step 3. Otherwise go to step 2.

2. If the new objective function value is lower than any of the previous values, then find the vector of differences (Δ_{tq}) between the new decision variables and those decision variables which resulted in the highest value of the objective function in the k previous experiments. Multiply Δ_{tq} by a step size ρ $(0 < \rho < 1)$ and add the result to each of the corresponding new decision variables. Return them to the simulation for another experiment. If still no improvement occurs, then reduce the step size (reduce ρ). Repeat until improvement occurs (in which case go to step 3) or until a prespecified number of step size reductions have taken place (go to step 5).

3. If improvement occurs, test the convergence criterion. Is the difference between the high and low values of the objective function found from the k simulations less than some prespecified ε? If yes, go to step 5; if no, go to step 4.

4. Using the k simulation results, fit the transition functions and find a new optimal policy (d_{tq}). Test this policy by simulation. Return to step 1.

5. Print results.

To handle terminal constraints in the calculation of the objective function from the simulation, a penalty-function technique is used. If U is the terminal value of the objective function calculated from the simulation, then the penalized objective (OBJ) is

$$\text{OBJ} = \begin{cases} U \text{ if constraints are met} \\ U-\phi \text{ if constraints are not met} \end{cases}$$

where ϕ is a number much greater than the maximum value which U may attain.

Computational experience shows that there are limitations to the application of the algorithm. First, a global optimum cannot be guaranteed because of the nonconvex nature of the response surface. Sometimes the program converges to local optima, but by using different starting points for the algorithm for each optimization, local optima may be detected. Also, care must be taken to be certain that at least one of the k original experiments meets all three of the terminal constraints. If all of the k initial data points were infeasible, the algorithm is incapable of establishing a proper trajectory toward a feasible solution.

Analysis of changes in parameter values after the optimization problem has been solved (postoptimality analysis) can be conducted in the case of linear models without performing another optimization problem.[5] However, because of the nonlinear, nonanalytical nature of our model, it was necessary for us to solve a new optimization problem with each change in parameter values.

RESULTS AND DISCUSSION

Application of the model to the Rich Creek decision problem requires some input information. In many cases this information was in the form of subjective estimation by the authors. However, validity checks helped to determine how useful the program is. One means of testing the validity of the model is to compare predicted outputs with outputs observed from the real system. In the Rich Creek ecosystem most of the system variables cannot actually be observed. However, two variables were observed:

The first was the catch of baitfish by seines that resulted from field studies in the summer of 1973. Using decision variables which were in effect in 1973, these field studies were mimicked by the model. Both deterministic and Monte Carlo predictions of the baitfish and crayfish catch by seine and trap were made (Table 3).

The second variable observed was the angling pressure in Rich Creek in 1973. West Virginia's Department of Natural Resources reported that Rich Creek angling pressure ranged from 300 to 500 angler-hours/acre/year. This corresponds to 2778-4630 angler-days/year. The simulation model gave a mean of 4719 angler-days/year for seven Monte Carlo simulations, which is comparable to the observed range. These results may be considered as indicative of the model's validity. However, the results are only comments on the validity of selected portions of the model and only under *existing* 1973 conditions. Because no guarantee can be made as to the model's ability to predict future events, it is imperative that relationships between variables be investigated in the analysis.

Table 3

Number of baitfish (caught) as observed and as predicted by the model

Time	Observed	*	Monte Carlo predictions (7 runs)						
May 16-31	164	113	120	107	22	36	79	81	43
June 1-15	182	114	184	84	478	39	82	102	37
June 16-30	148	111	128	50	413	178	68	57	535
July 1-15	204	109	912	56	307	39	152	50	106
July 16-31	191	107	257	52	335	21	56	47	32
Aug 1-15	319	106	310	47	266	22	34	55	34
Aug 16-31	276	104	81	49	224	19	42	48	33
Sept 1-16	244	102	75	45	20	17	33	24	14
Totals	1728	866	2067	496	2065	371	546	464	834

* Deterministic prediction mean = 977.5

For the initial optimization problem, the terminal constraints were set as follows:

$$\text{BDG} = \$6,960 \geq B$$
$$\text{COM} = 5,000 \leq C$$
$$\text{DV} = 100,000 \leq D$$

The optimal value of utility to anglers (U) derived from this program was 7.668 units of fishing satisfaction per day. The commercial catch, budget expenditures, and diversity in this prediction were budget B = \$6,512, commercial benefits (fish caught) C = 5,701, and diversity D = 494,300. As we shall see, the system was not greatly constrained under these terminal conditions.

Experimenting with the model shows that the approximate maximum level of cumulative commercial catch per seine-day which can be reached is 6,000 fish, while the maximum diversity D is 10^6. These figures are dependent upon input variables (primarily the estimates of initial population sizes). Subjective estimates are made for many of these variables, but these outputs serve as reference points for the postoptimality analysis.

In investigating the sensitivities of the terminal constraints, the first situation tested was: BDG = \$50,000 (seven times the budget for the initial optimization reported above), COM = 0, and DV = 0. In effect, this is the problem of *unconstrained* maximization of utility, and the resulting optimal utility U^* was 8.891. This U^* is achieved for B = \$26,044, C = 5,692, and D = 101,400, showing the diminishing returns for dollar expenditures. Nearly quadrupling the budget increased the average angler's satisfaction by about one-sixth, and the commercial catch decreased very slightly. At the same time diversity decreased by about four-fifths. These three numbers partially define the bounds of the constrained region. If the terminal constraints are within these bounds, these constraints will not greatly affect the solution.

The feasible region bounded by commercial catch COM and diversity DV is a function of the interaction of these two components. If COM is high, then diversity cannot reach as high a level as when COM is low. Conversely when DV is high, commercial catch cannot reach as high a level as when it is low. This means that under most circumstances the constraints of commercial catch and diversity are not tight unless the region is rather narrowly defined.

When the shape and structure of the objective function was originally defined, it was found that (all else being equal) the angler received more satisfaction (utility) from a unit of trout than from a unit of either bluegill or smallmouth bass. Therefore, when the budget was high, more money could be spent, more trout could be planted, and the maximum possible angler's satisfaction U^* primarily reflected the utility of trout fishing. But, when B was low, not as many trout could be planted and, thus, the other types of angling took on increased importance. The optimal decisions and the role of other constraints are, themselves, affected by this switch in importance.

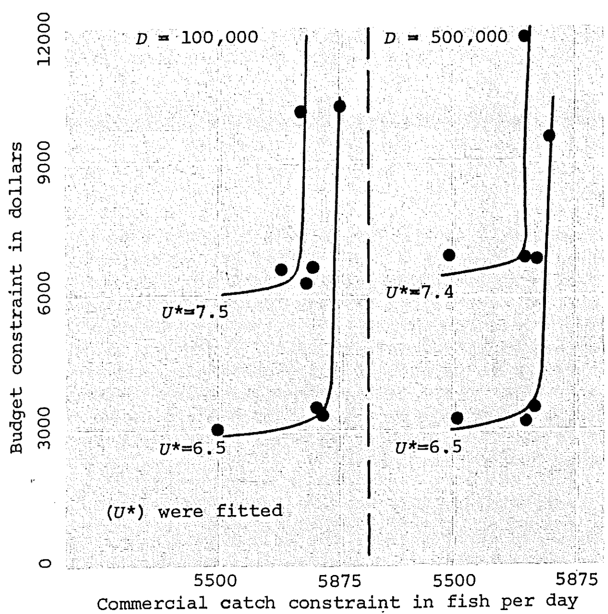

Figure 1 - Commercial catch constraint in cumulative catch per seine-day versus budget constraint, given two values of the diversity constraint, DV = 100,000 on the left and DV = 500,000 on the right. Contour lines of equal optimal utility (U^*) were fitted by eye

When the budget constraint BDG is approximately \$3,500, the optimal solution occurs when diversity is greater than 5×10^5; i.e., decreasing DV to 10^5 the diversity constraint does not increase U^* (Figure 1). But when BDG is high (greater than \$6,000) a decrease in DV to 10^5 increases U^*. Also, for fixed U^* and fixed B (e.g., $U^* = 6.5$ and $B = \$4,500$), the commercial catch C is higher when DV = 10^5 than when DV = 5×10^5.

The analysis indicates that a high value of the diversity index D occurs at low budget expenditures B. This implies that a more diverse system is needed when the natural portions of the ecosystem (smallmouth bass and bluegill) are being exploited at a higher rate. An increase in diversity may provide more opportunities for the remaining species to grow and reproduce. Conversely, at high budgets the maintenance of high diversity will result in an indirect output loss from the trout portion of the ecosystem since the concentration of trout is unfavorable to other species. Therefore at high budgets, a high diversity can only occur when angler's satisfaction (utility U) is lower; i.e., diversity becomes an active constraint.

Since utility reflects the preferences of the angler, sensitivity analysis should especially include those variables and parameters which affect the man-biota interaction. Human population size (POP) has a negative effect on U^* (Figure 2). This is a logical situation because one attribute in the objective function U is privacy. Increasing the population means more people will fish, yielding less privacy and fewer fish per angler.

The aquatic fauna in the ecosystem are affected primarily by two major factors: (1) the water temperature and (2) the standard metabolic rate of the fish and crayfish. When the entire temperature function is shifted 2°C higher, the optimal utility of 8.112 is achieved with D = 809,700. When the temperature function is shifted 2°C down, the diversity constraint can not be met at D = 100,000, the minimum acceptable value of DV; hence no solution is possible.

When the standard metabolic rate is reduced 10% for all fish and crayfish, the effect on angler satisfaction is not great ($U^* = 7.454$). However, when the metabolic rate is *increased* 10%, once again, the diversity constraint can not be met at D = 100,000, and no solution exists.

Temperature and metabolic rate, though related, operate differently in the ecosystem: When temperature increases, it increases the metabolic rate. However, it also increases the swimming speed; hence the animal is more likely to be successful as a predator. This gain apparently outweighs the energy losses to higher metabolism. The gain is reflected in faster growth, more reproduction, and subsequently a gain in diversity *and* angler satisfaction (utility). On the other hand, increasing the standard metabolic rate alone without an associated increase in predatory efficiency results in a net energy loss, a subsequent reduction in diversity, and also an increase in utility.

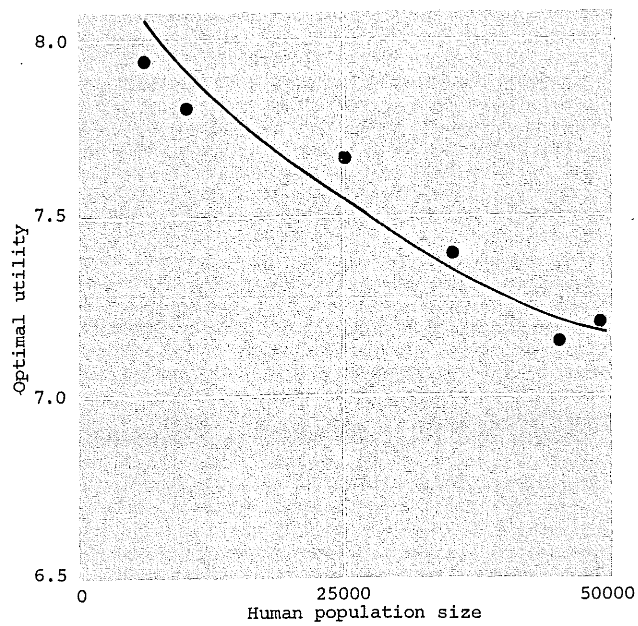

Figure 2 - Optimal cumulative utility obtained from various human population sizes; curve fitted by eye

Another question (particularly relevant to the practicing ecosystem manager) asked is *How important are the initial estimates of the fish population?* This question was investigated by (1) halving and (2) doubling all of the initial population estimates. It must be remembered that doing this alters the diversity index. The resulting problem was somewhat circumvented by (1) doubling and (2) halving the population sizes at the end of each simulation *before* the diversity was calculated. In this way, differences between diversities were suppressed, so that the same diversity constraint could be used in both cases. The results showed that doubling the initial population estimates produced $U^* = 7,584$, while halving them produced $U^* = 8.381$. Considering the artificial way in which the diversity constraint was suppressed, these differences do not seem significant.

CONCLUSIONS

Postoptimality analysis of the stream management simulation model shows the interaction of the budget, commercial catch, and diversity constraints and the effect of this interaction on optimal satisfaction for anglers (utility). At high budgets most of the recreational fishing is for trout, and the diversity constraint keeps utility from achieving higher values. At low budgets, other fish species are exploited by recreational anglers, and diversity is high without requiring high values of the diversity constraint. The commercial catch is inversely correlated with diversity, but it has little effect on utility.

Variables which produced large changes in utility U when perturbed are budget, water temperature, number of anglers, fish metabolic rate, and the biomass of trout. The study shows that to maximize U, management should try to increase the number of trout planted and to disperse the fishing pressure throughout the year.

Research to improve the model of Rich Creek and subsequently to understand the fishery system better should include investigations of temperature changes over time and of the metabolism of the fish (especially trout).

Postoptimality analysis of the simulation model of Rich Creek was a valuable exercise. Even though parameter and inputs were subject to large estimation errors, several significant results were achieved: (1) a strategy for optimizing angler satisfaction with acceptable constraint, (2) the relation of this strategy to parameters and constraints, and (3) perhaps more significantly, identification of those variables which have the largest effect on the objective function and which are therefore the key to planning future research.

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DISCUSSION

Cale: Would you explain the meaning of $1/P_{oi}$ with reference to its time units and its relation to P_{oi}?

Powers: A steady-state probability in terms of a Markov chain may be interpreted as the proportion of time that is spent (in the long run) in a certain state [P_{oi} = proportion of time spent in steady state i]. When this value is divided into 1 unit of time, the quotient is the number of time units that are expected to pass before the return to this state (expected recurrence time). The time units are actually the number of transition stages in the Markov chain which may be equivalently defined in terms of time (1 transition stage is equivalent to 1 hour of interaction time), i.e., continuous time is broken into discrete time steps of 1 hour.

Cheslak: You identified a certain fixed priority schedule of activities and some probability functions based upon these activities. Animals seldom act in terms of a fixed time-invariant priority schedule; therefore, what changes would occur in your postoptimality study if you introduced a nonfixed time-variant priority schedule?

Powers: Each of the steady-state probabilities (probabilities of being in the states of serving an item of priority i and probabilities of being idle having successfully completed a service of priority i) are functions of the arrival rates. At different times of the year, the proportion of animals which will fall in each priority category will vary. Therefore, the arrival rates were weighted by estimates of these time-dependent proportions. These proportions of arrivals would be ignored by the animal and, therefore, from the animal's point of view, they were not arrivals at all.

Clark, W.C.: In deriving your overall objective function U from its component parts (U_1, U_2, U_3, U_4), at least three judgments must be made: (1) Are the values assigned to any one of the components independent of values taken by other components at the same time; that is, is a certain quantity of privacy valued equally, independent of the number of fish being caught?

Powers: The attributes of U were formulated and scaled in such a way that the assumptions of preferential and utility independence were valid. These assumptions were validated by discussions with fishermen.

Clark, W.C.: The second judgment to be made is as follows: (2) Are the values of the individual or aggregate U's time-independent; that is, are five fish valued as highly early in the season as later? (This is the discount problem.)

Powers: With the short time-horizon (one year), discounting was assumed to have minimal effect. The preference of the fishermen does not appear to change.

Clark, W.C.: The third and final judgment to be made is as follows: (3) Are the temporal patterns of the U's important, even where the answer to (2) is "yes" and the average value of the U is the same, independent of pattern? In other words, is the time series of fish catches (0,6,0) valued equally with one of (2,2,2)? (Does the rate of change or autocorrelation matter?) The answers to these questions are not *a priori* obvious in any given instance, and the way in which we answer them can seriously influence the behavior of the aggregate U. Several studies of preferences in the design of resource systems have shown the answers to often be "no," "no," and "no." Were these answers derived from your questionnaire, or were they adopted by your team as (hopefully) reasonable assumptions?

Powers: We assumed that disutility associated with the rate of change of these attributes was nonexistent. It was for precisely that reason that several attributes were considered in U. If one attribute was low, often the other attributes would dominate. Therefore, the rate of change in U would be relatively small, and the disutilities, if they existed, would be artificially masked.

Clark, C.W.: In response to Bill Clark's question: first, it seems that intertemporal preference would be of little importance in your case because of the one-year time horizon. Second, time-varying parameters should be easily dealt with. Disutility associated with fluctuations, however, could introduce nontrivial nonlinearities into the optimization model.

Powers: Our assumptions of nonfluctuating preferences appeared to be good for the population with which we were dealing. However, one of the reasons for using a short time-horizon (one year) was for the purpose of reevaluating all inputs at the end of each year, and this is what we recommend.

Crow: After you do your initial five runs and optimize along the curve, you go back with another simulation run. Is this just to test your optimization? If your solution is not optimal, what do you do? How does this differ from a steepest-descent method?

Powers: You derive an optimal decision policy for the approximating models fitted to the initial k simulation experiments ($k = 5$ for our applications). Then apply this policy to the next simulation run and test the results of its use in the simulation. The result of this test will not in general be optimal; it will only produce a better value of the objective function or a worse value. If it is a worse value, you adjust step sizes. If it is better than the worst value of the objective function as calculated in the k simulation experiments, then replace the state and decision variables of the improved objective function. Convergence occurs if all k objective-function results become sufficiently close together.

To use the gradient method, you would need estimates of the first derivatives. To get these estimates for a system with the number of decision variables with which we were dealing would be computationally infeasible.

Smith, F.E.: Your subroutines are very detailed. Have you examined them to see if all of the behavior patterns and details of age structure are needed; that is, do all of them have significant effects on the output? Can the model be simplified?

Powers: We have not asked to see if the details in *structure* produced significant effects. However, we did find that the estimates of most of the parameters associated with this detail were relatively insignificant. Therefore, the details, which are logically appealing (especially when they are presented to nonsystems ecologists), may not be important, but they provide a framework for applications to other ecosystems in which these components might be significant.

Cheslak: In your conclusion you implied that simulations of natural ecosystems could be benefited by the definition of explicit objective functions. Are you implying that a natural ecosystem has objectives, and if so could you identify some for me?

Powers: I believe that I implied that *analysis* of simulations of natural ecosystems would be benefited by explicit objective functions, i.e., explicit objective functions are equivalent to clearly stated hypotheses in empirical studies, and simulation experiments should provide information in relation to these hypotheses.

Simpson: For whom was the project done, and did they use the results?

Powers: This study was funded by the West Virginia Department of Natural Resources, and the results are for their use. The results reported here are preliminary results; a more complete report will be forwarded to West Virginia D.N.R. Discussions with representatives of West Virginia have shown they are interested in the conclusions, especially conclusions about the structure of the optimal decisions, marginal utilities, and effects of perturbing the constraints. The computer program was designed so that it could be easily adapted to management of any aquatic ecosystem. Therefore, it could be used by state, federal, or private agencies interested in determining optimal decisions for managing their aquatic system.

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